MOEA Evaluation Metrics

CS454 AI-Based Software Engineering

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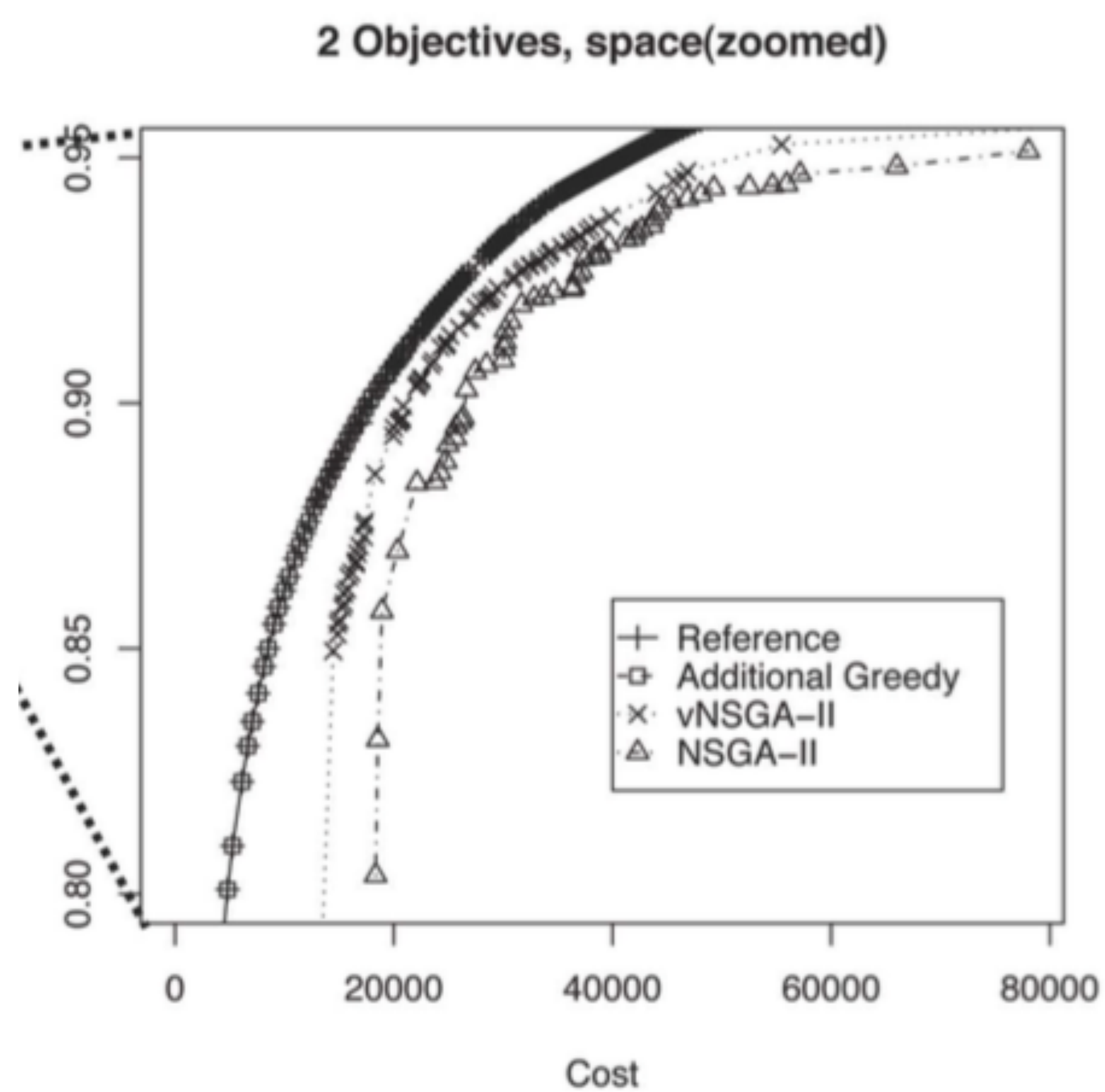
Comparing Pareto Fronts

- Comparison itself is not as straightforward as comparing two scalar values.
- There is no reference point, as the true Pareto front is usually not known.



Empirical Evaluation

- Empirical evaluation of different MOEAs becomes a meta-comparison: it is not only about the domain-specific quality, but it is also about the quality of the front itself.
- Properties that we want to evaluate:
  - Closeness to the true Pareto front
  - Diversity of the solutions on the Pareto front

Closeness to true front

- There are cases where the true Front is known:
  - for example, benchmark optimisation problems.
- For cases where the true front is not known, we use what’s called “reference front”:
  - Collect all known solutions from all MOEAs involved.
  - Extract a single Pareto front from the collected solutions.
- Reference Pareto Front will include solutions contributed by different MOEAs.

Generational Distance (GD) and Inverted Generational Distance (IGD)

- GD: average distance from each solution to its closest reference point.
- IGD: average distance from each reference point to its closest solution
- The smaller, the better.

Weaknesses of GD

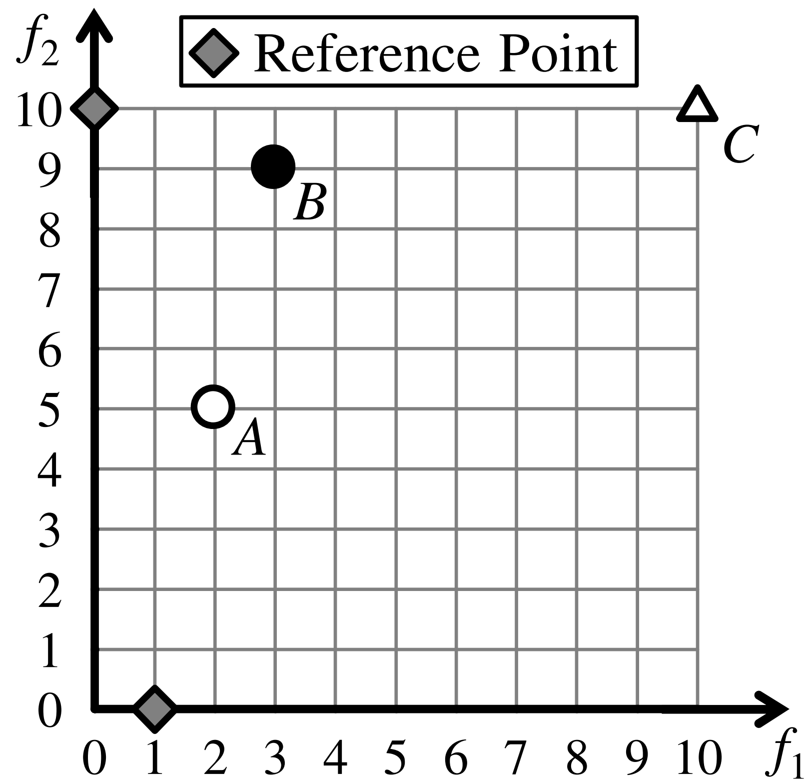


Fig. 1. Example 1 (Zitzler et al. [26])

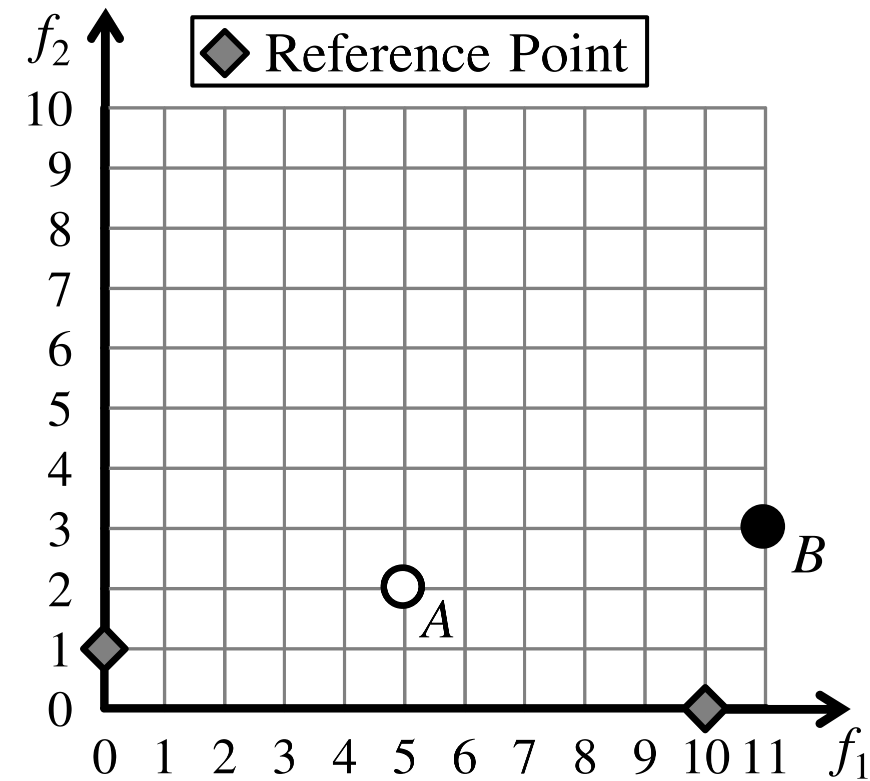


Fig. 2. Example 2 (Schütze et al. [18])

B has the shortest distance to its closest reference point,
but arguably A is a better solution.

Modified Distance Calculation in Generational Distance and Inverted Generational Distance, Hisao Ishibuchi, Hiroyuki Masuda, Yuki Tanigaki, Yusuke Nojima, EMO 2015.

Weaknesses of GD

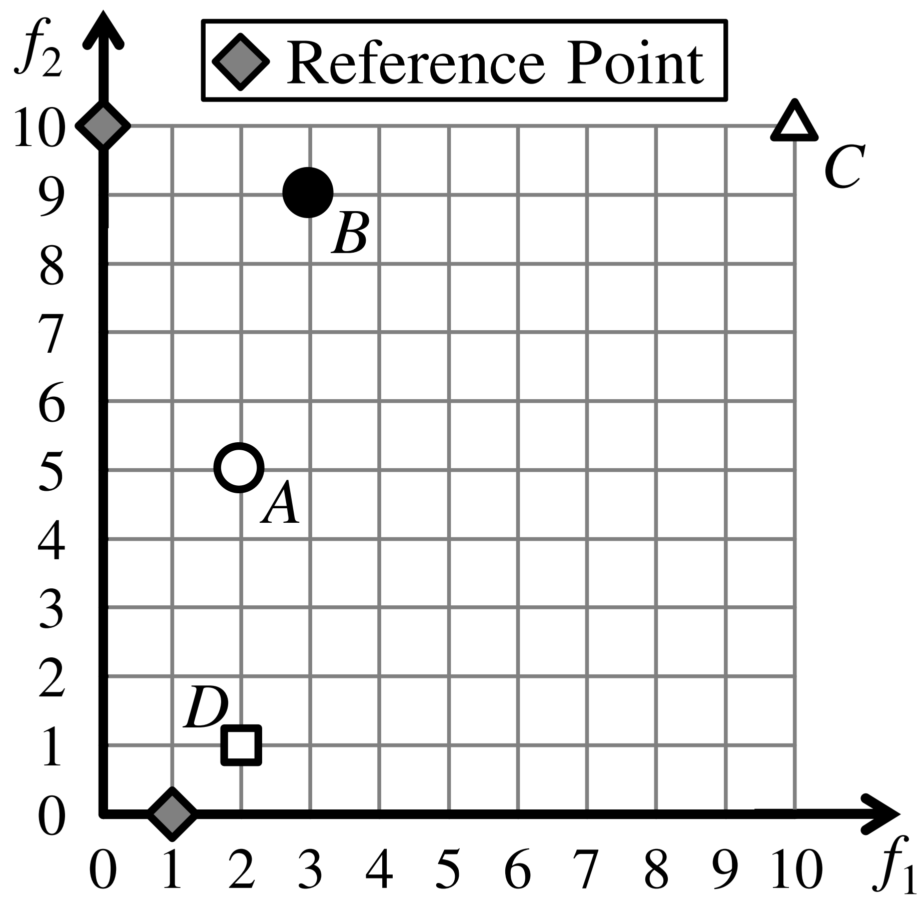


Fig. 3. Example 3 with a new solution set D

IGD is more sensitive to gaps.

$$IGD(A) = \frac{1}{2}\left(\sqrt{(2-0)^2 + (5-10)^2} + \sqrt{(2-1)^2 + (5-0)^2}\right) = 5.24,$$

$$IGD(D) = \frac{1}{2}\left(\sqrt{(2-0)^2 + (1-10)^2} + \sqrt{(2-1)^2 + (1-0)^2}\right) = 5.32.$$

Modified Distance Calculation in Generational Distance and Inverted Generational Distance, Hisao Ishibuchi, Hiroyuki Masuda, Yuki Tanigaki, Yusuke Nojima, EMO 2015.

Weaknesses of GD/IGD

- When the shape of the true Front is known as a continuous function: reference Pareto front, i.e. a set of points, is sampled from the function.

- How you sample can affect distances

    - For 11 reference points, IGD(Z, A) < IGD(Z, B)

    - For 21 reference points, IGD(Z, A) > IGD(Z. B)

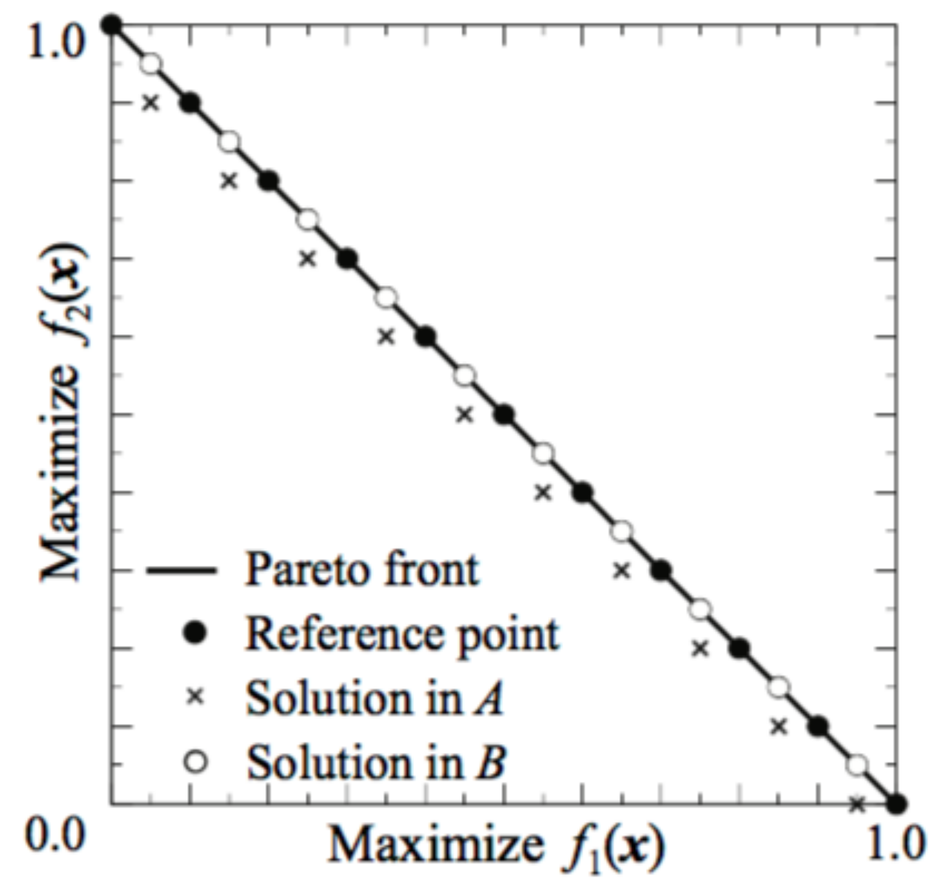


Fig. 3. Reference points ($H = 10$) and two solution sets A and B.

H. Ishibuchi, H. Masuda, Y. Tanigaki, and Y. Nojima. Difficulties in specifying reference points to calculate the inverted generational distance for many-objective optimization problems. In Computational Intelligence in Multi-Criteria Decision-Making (MCDM), 2014 IEEE Symposium on, pages 170–177, Dec 2014.

Weaknesses of IGD

- When using collected reference front, different MOEAs will contribute solutions with different characteristics:
  - some may show strong convergence
  - others may show greater diversity
- Again, this may results in sampling bias when evaluating an MOEA.

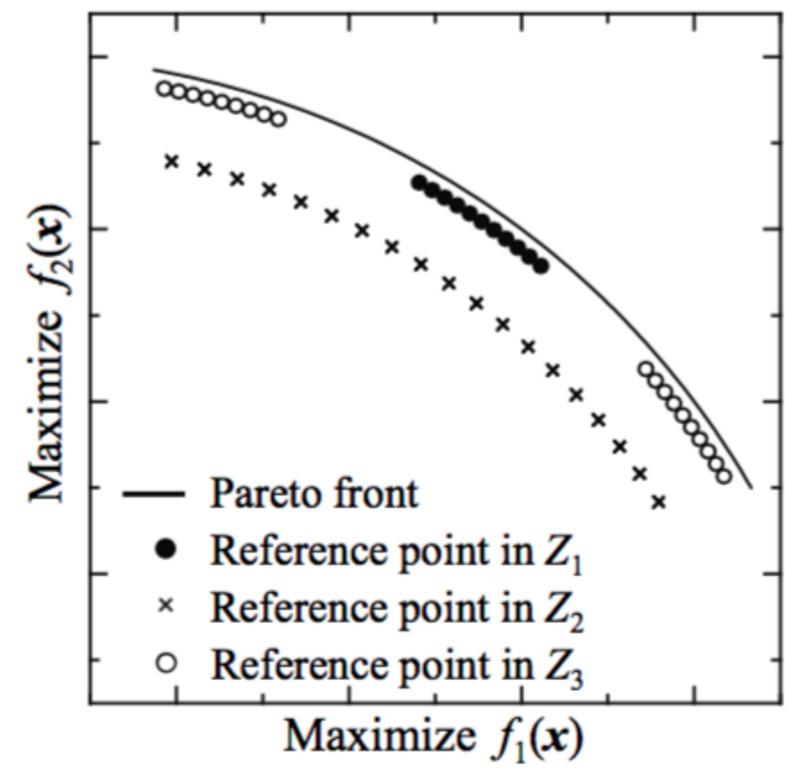


Fig. 7. Three typical situations of reference points: Concentration on the center of the Pareto front (Z_1), uniform distribution over the entire Pareto front (Z_2), and emphasis on the edges of the Pareto front (Z_3).

H. Ishibuchi, H. Masuda, Y. Tanigaki, and Y. Nojima. Difficulties in specifying reference points to calculate the inverted generational distance for many-objective optimization problems. In Computational Intelligence in Multi-Criteria Decision-Making (MCDM), 2014 IEEE Symposium on, pages 170–177, Dec 2014.

Epsilon

- Binary indicator, $I_\varepsilon(A, B)$: intuitively, the amount of "shift" required to change B so that it is weakly dominated by A (i.e. A is not worse than B in all objectives).

$$I_\epsilon(A, B) = \inf_{\epsilon \in \mathbb{R}} \{\forall z^2 \in B \exists z^1 \in A : z^1 \preceq_\epsilon z^2\}$$

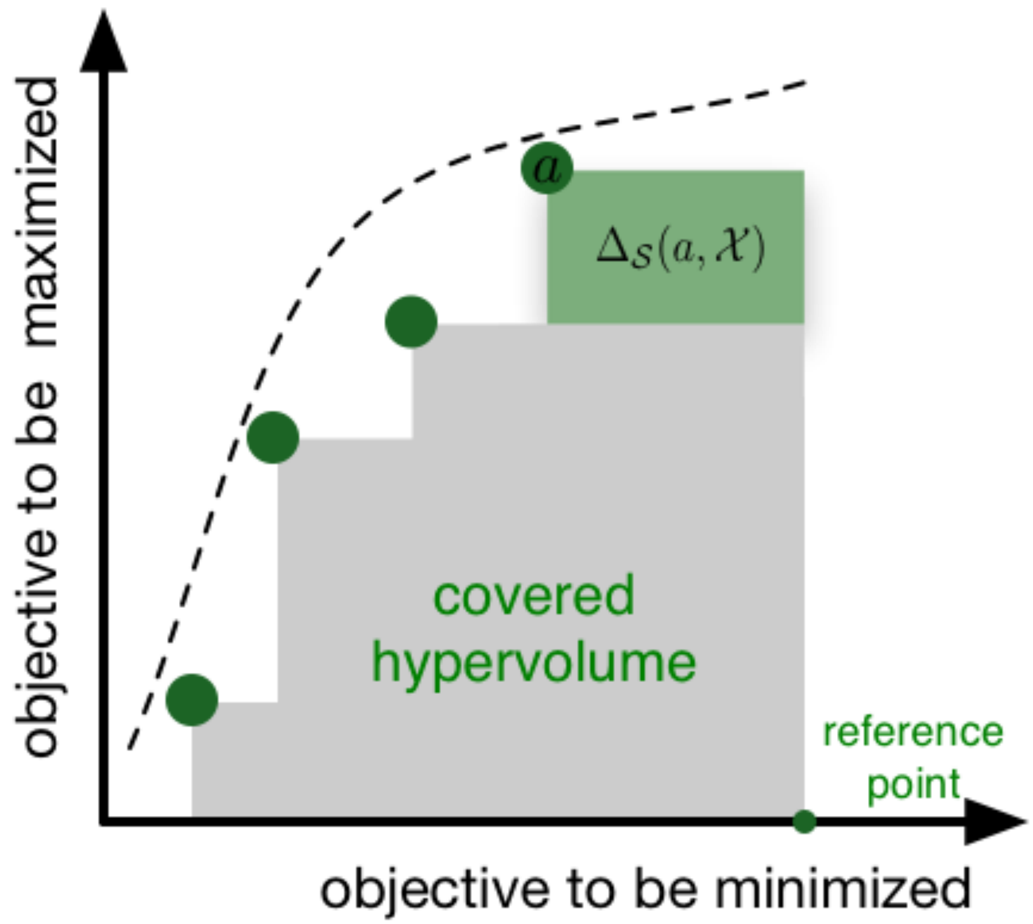
$$\boldsymbol{z}^1 \preceq_\epsilon \boldsymbol{z}^2 \iff \forall i \in 1..n \ : \ z_i^1 \leq \epsilon \cdot z_i^2$$

$$\boldsymbol{z}^1 \preceq_{\epsilon+} \boldsymbol{z}^2 \iff \forall i \in 1..n \ : \ z_i^1 \leq \epsilon + z_i^2.$$

$$I_\epsilon^1(A) = I_\epsilon(A, R),$$

Hypervolume

- Intuitively, hypervolume measures the area (space) dominated by a given Pareto front.
- An unary indicator: does not need a reference front.
- Can be thought to measure both convergence and diversity.



Scaling and Normalisation

- The concept of Pareto optimality itself is independent from scale and normalisation: it is strictly based on partial order only.
- For quality indicators, normalisation may be necessary:
  - so that multiple objectives contribute equally to the indicators.

Simple Linear Scaling

- … may be applicable, if the bounds are known.
- If not, there are other normalisation methods.
- … which leads into the next topic :)

$$z_i' = \frac{z_i - z_i^{min}}{z_i^{max} - z_i^{min}}$$